

Infant, Junior School and Camp Education

Progression in Calculation Policy

2019

This document explains how to teach calculation with understanding, and not just as a process that is to be remembered. The Written Calculation Policy clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports the development of mathematical concepts.

The Aims of the Curriculum:

The National Curriculum for Mathematics aims to ensure that all children:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practise with increasingly complex problems over time, so that children develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Aims of the Policy:

- To ensure consistency and progression in our approach to calculation and enable a smooth transition between year groups and phases.
- > To ensure that children develop an efficient, reliable, formal written method of calculation for all operations.
- > To ensure that children can use these methods accurately with confidence and understanding.
- > To ensure children understand important concepts and make connections within mathematics.
- > To ensure children show high levels of fluency in performing written and mental calculations.
- To ensure that children are ready for the next stage of learning and have been given strong foundations in mental methods, the use of practical equipment, allowed to explore jottings in a range of forms and then to move onto more formal recording using a strong knowledge of place value, number lines labelled or blank, partitioning before eventually using compact written methods.
- > To ensure that children are competent in fluency, reasoning and problem solving and can make informed and appropriate choices about the methods they wish to use (mental or written) to solve mathematical problems efficiently and effectively.

Introduction

The policy is set out in operations, addition, subtraction, multiplication and division. Within each specific area there is a progression of skills, knowledge and layout for written methods. The calculation strategies which will be used will reflect this ideology – moving from concrete to pictorial and then abstract recording leading to more formal written methods. Mental methods and strategies will work in partnership with these methods.

It is important to always show the links between operations and not teach them in isolation or without showing, in practical problem solving activities and across all mathematical topics, how these operations can be applied.

It is important that staff always use correct mathematical language and encourage this from every pupil. This will take place in class discussions as well as through oral and written feedback, next steps and target setting.

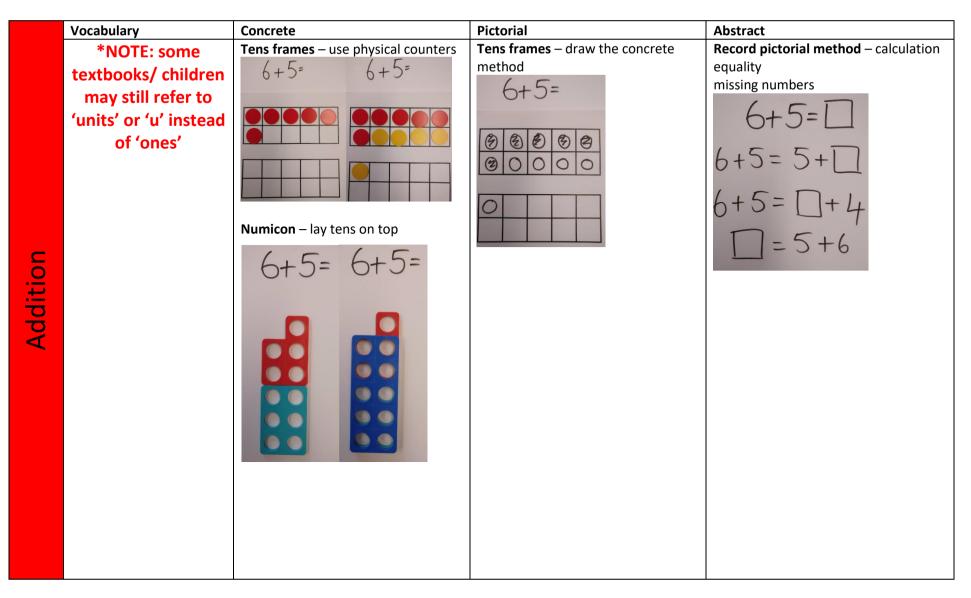
We have chosen not to identify which year group should use which method because we wish staff and children to have the freedom to take the next steps on their mathematical journey when they are ready to do so and if the policy is followed, there should not be a problem with progression as children move through the school.

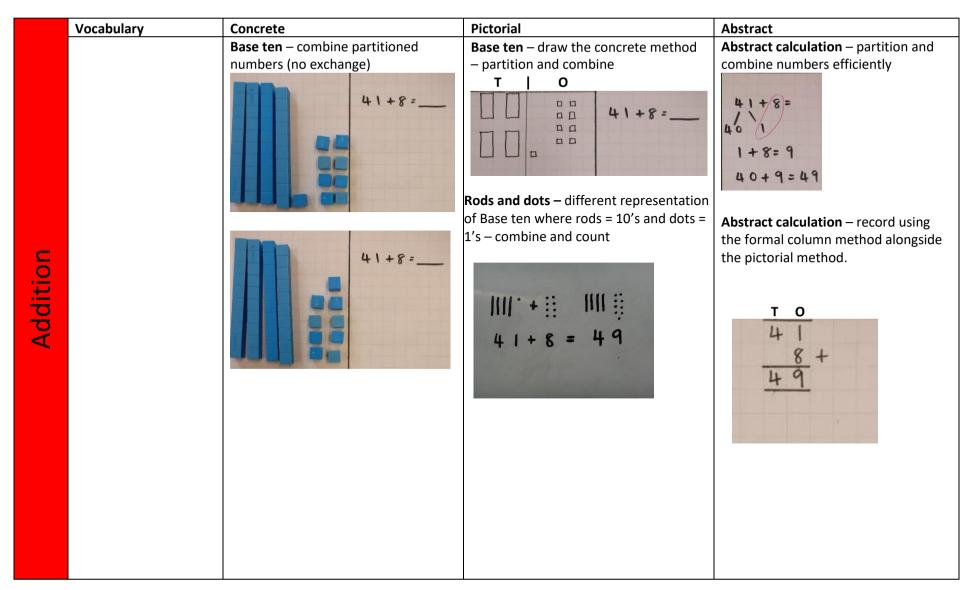
All staff have the responsibility to make sure that children have the depth of knowledge and experiences required to move onto the next stage of their development rather than pushing them on too quickly. This leads to misconceptions and poor mathematical foundations and eventually, in later years, children will not be able to make the required progress.

Ultimately we aim to enable children to make informed choices about the methods they use both mental and written that are the most efficient and this includes recognised compact methods.

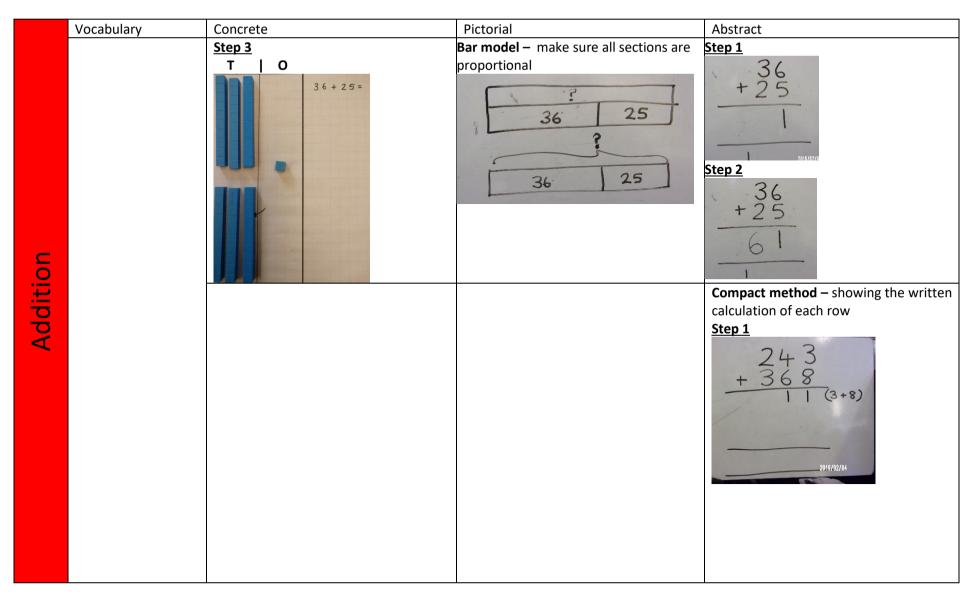
Addition

Vocabulary	Concrete	Pictorial	Abstract
abstract	PPW – use manipulatives to put two	PPW – draw the concrete method	PPW – record the pictorial method
addition	parts together	$ \begin{pmatrix} 0 & 0 \\ 0$	using numbers
altogether			
base ten			
bead		1 (00) 1000	
strings			(9) (5)
calculation			0
combine))	
concrete			
counters	Number track – count on from one	Number track – count on in ones	Empty number line – ENL – count on
ENL – empty number line	of the numbers	000	(in ones or chunks)
equal	4+3=		13
exchange		3 (4) 5 6 (7) 8 9	
hundreds			
multilink			
numicon number line		Marked number line –count on in	45618
	Bead String – making groups of beads	ones or chunks	
	and then counting them altogether	\sim	
partition			+3
pictorial		2 5 4 5 6 0 8 1 10	
place value	4 + 3		1 1
PPW – part-part-whole	↓	Bar model – comparison and	4 7
record	1	showing parts of a whole	
sum		4]]	
tens	Numicon –find the piece that fits		
tens frame		3	
thousands			
total	1		
		?	
	0000	4 3	



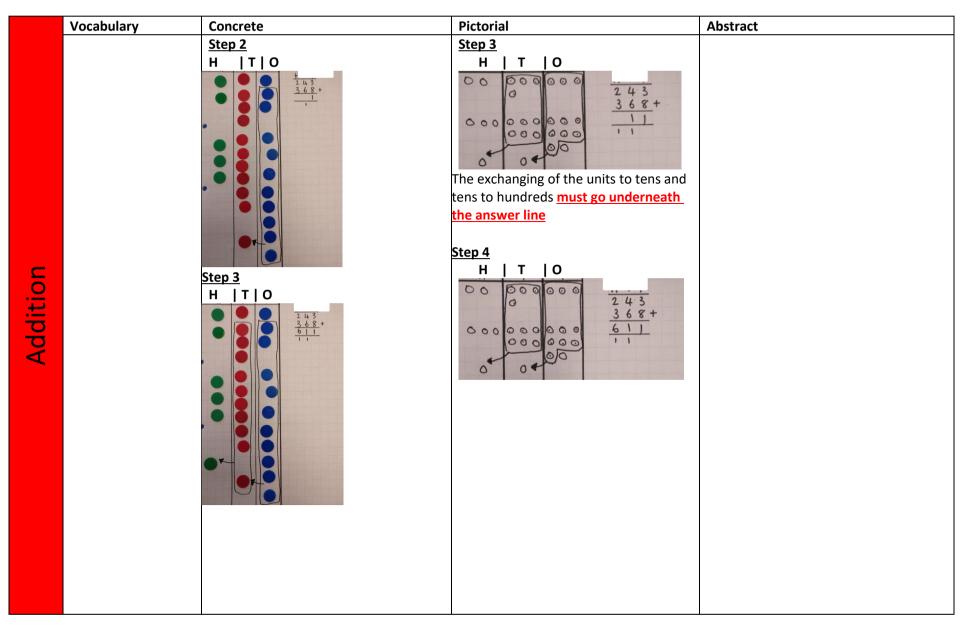


Pictorial Vocabularv Concrete Abstract Base ten – combine partitioned Base ten – draw the concrete method Abstract calculation – record the sections and **exchange** when you - partition, combine and exchange. formal column method alongside the make ten. Children need a strong pictorial method. understanding that 1 ten is the same 0 Step 1 Т 36 25+ 61 as 10 ones. Т 0 100 36+25= Step 1 т 0 36+25= Ē 000 00 Step 2 0 т Addition 100 36+25=61 000 Abstract calculation – partition and combine like terms (add the ones together and add the tens together) 100 36+25= 20 Step 2 т 0 30+20=50 6+5=11 6 36+25= 50+11=61 **Rods and dots** – where there are more than 10 dots, exchange 10 dots for a rod which equals a 10 36 + 25 = ∬(<u>;</u>;) = ||||||| 6



	Vocabulary	Concrete	Pictorial	Abstract
Addition	Vocabulary	Concrete	Pictorial	Abstract Step 2 243 +368 100(60+40) 2015/02/4 Step 3 243 +368 11(3+8) 100(60+40) 500(300+200) 2015/02/4 Step 4 243 +368 11(3+8) 100(60+40) 500(300+200) 2015/02/4

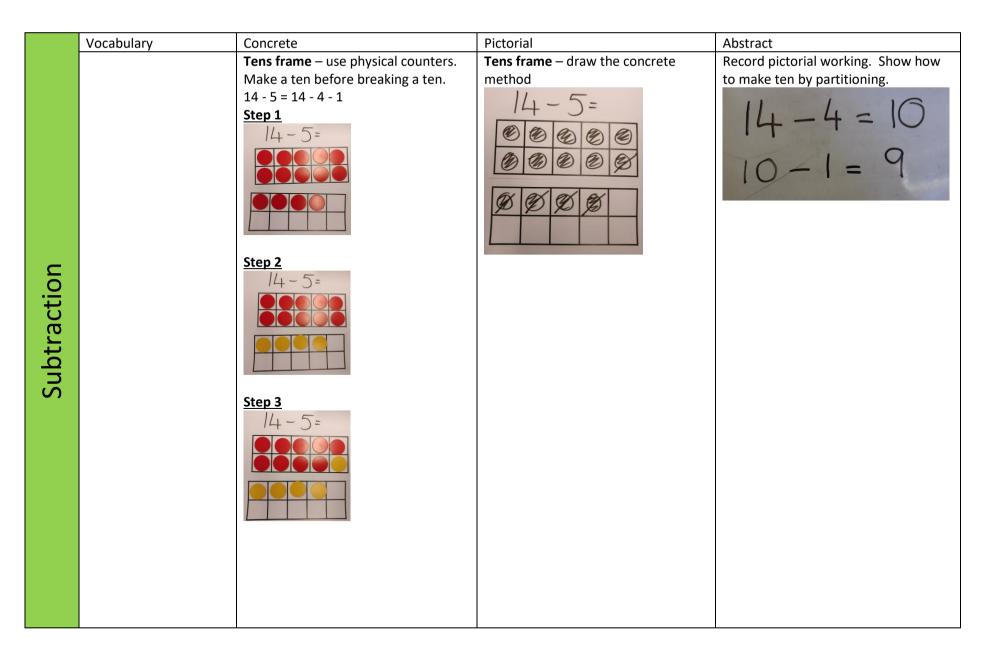
	Vocabulary	Concrete	Pictorial	Abstract
Addition				$\frac{5 \text{tep 5}}{243} + 368 + 368 + 11 (3+8) + 100 (60+40) + 500 (300+200) + 500 (300+200) + 500 $
Add		Place value counters (either use colour coding or labelling) – partition, combine and exchange. Step 1 H T O	Place value counters – draw the concrete method – partition, combine and exchange. Step 1 H T O 2 4 3 3 6 8 + 3 6 8 + 3 6 8 + 3 6 8 + 2 4 3 3 6 8 + 1 2 4 3 3 6 8 + 1 1 2 4 3 3 6 8 + 1 1 1 1 2 4 3 3 6 8 + 1 1 1 1 1 1 1 1	Abstract calculation – record the formal written method eventually without the pictorial method alongside $H \mid T \mid O$ $\boxed{2 \ 4 \ 3}$ $\underbrace{3 \ 6 \ 8 \ +}$ $\underbrace{6 \ 1 \ 1}$ $1 $

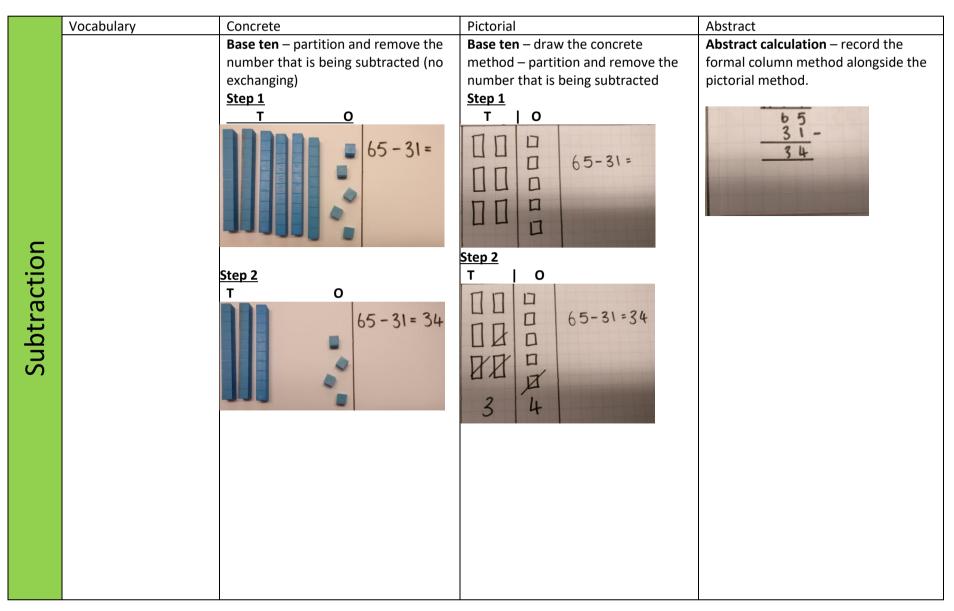


	Vocabulary	Concrete	Pictorial	Abstract
	abstract	Manipulatives – make the number	Draw the concrete method – draw	PPW – record numbers
	base ten	and remove what is being subtracted	the number and cross out what is	-
	bead	Step 1	being subtracted	(1)
	strings	4-3=		(4)
	calculation	+)	0000 4-3=	X
	concrete		OBBB 4-3=	
	counters			a' va
	decrease			(2) ()
	difference	Step 2		(S)
	equal	4-3=		\bigcirc
	exchange	+)		
	fewer	the second second second second		
<u>.</u>	hundreds			
	less			
a(minus			
tr	multilink			As above
<u>.</u>		missing piece?	What is needed to make the whole?	
л С	number line			
0)	number track		(00)	
	ones			
	partition		$\left(00\right)$	
	pictorial			
	place value	AA	AA	
	PPW – part-part-whole		() (00)	
	record			
	subtract			
	subtrahend			
	take away			
	tens			
	tons thousands total units*			

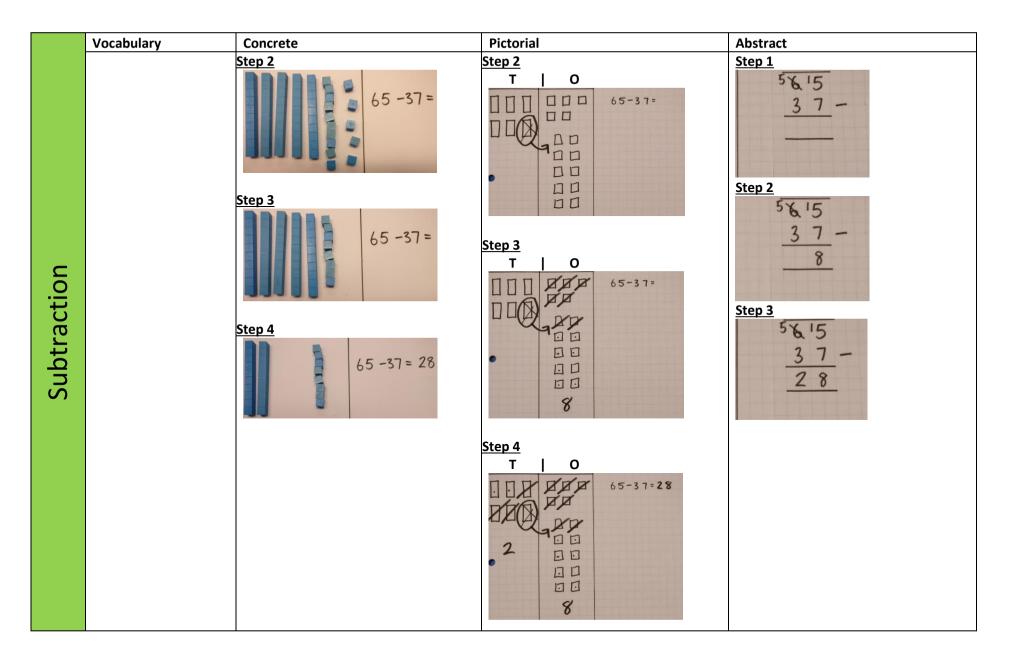
Concrete Pictorial Vocabularv Abstract Number track – use a counter to **Empty number line** – ENL – count Number track – draw the concrete physically count back method – count back in ones back (in ones) 4-3= 4-3= 1 2 3 4 5 6 7 8 9 10 11 1 2 3 4 5 6 7 8 9 10 11 12 13 14 Marked number line – draw the 1 2 3 jumps – count back in ones or chunks 4-3= Subtraction Bead String – count the correct number of beads then slide/count the beads to physically remove the subtracting number Step 1 4-3= Step 2

	Vocabulary	Concrete	Pictorial	Abstract
		Difference – use manipulatives Make both numbers and lay them next to each other, what is the difference? 4 - 3 =	Difference – draw the concrete method. What is the difference in length?	Difference – find the difference between 8 and 5 9 and 6, 8 and 5, 7 and 4 – why do they have the same difference?
Subtraction			Bar model – comparing to parts and finding the difference	





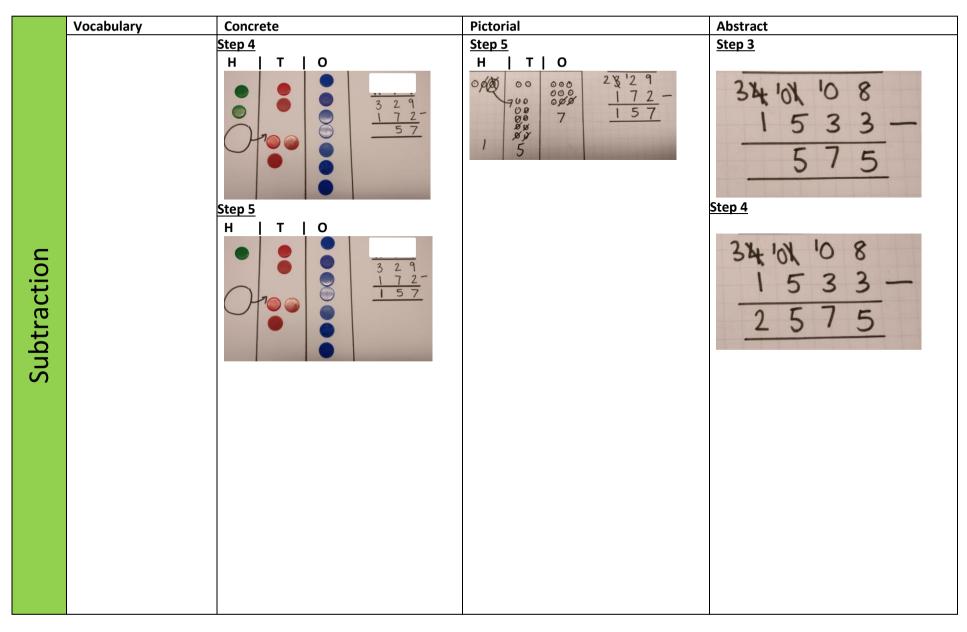
	Vocabulary	Concrete	Pictorial	Abstract
			Rods and dots – draw the first number	
			and cross out the number being	
			subtracted	
			<u>Step 1</u>	
			65 - 31 =	
uo			Step 2	
Subtraction			65 - 31 = 34	
ubtr				
S			The second second	
		Base ten – partition and remove the	Base ten – draw the concrete	Abstract calculation – record the
		number that is being subtracted,	method. Partition, remove the	formal column method alongside the
		exchanging when necessary.	number that is being subtracted and exchange where necessary.	pictorial method. Children must understand that we still
		<u>Step 1</u>	exchange where necessary.	have 41 when we exchange because
			<u>Step 1</u>	41 = 30 + 11
		65 -37 =	т О	



	Vocabulary	Concrete	Pictorial	Abstract
			Rods and dots – where exchanging of	
			the tens rod needs to occur, cross this	
			out and exchange into 10 dots 'ones'.	
			Then cross out the number being	
			subtracted and re-draw the reminding	
			rods and dots to provide the tens and	
			ones. Stop 1	
Subtraction			$\frac{\text{Step 1}}{65 - 37}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$ $\frac{1}{100}$	
			Step 3	
			65-37 = 28	

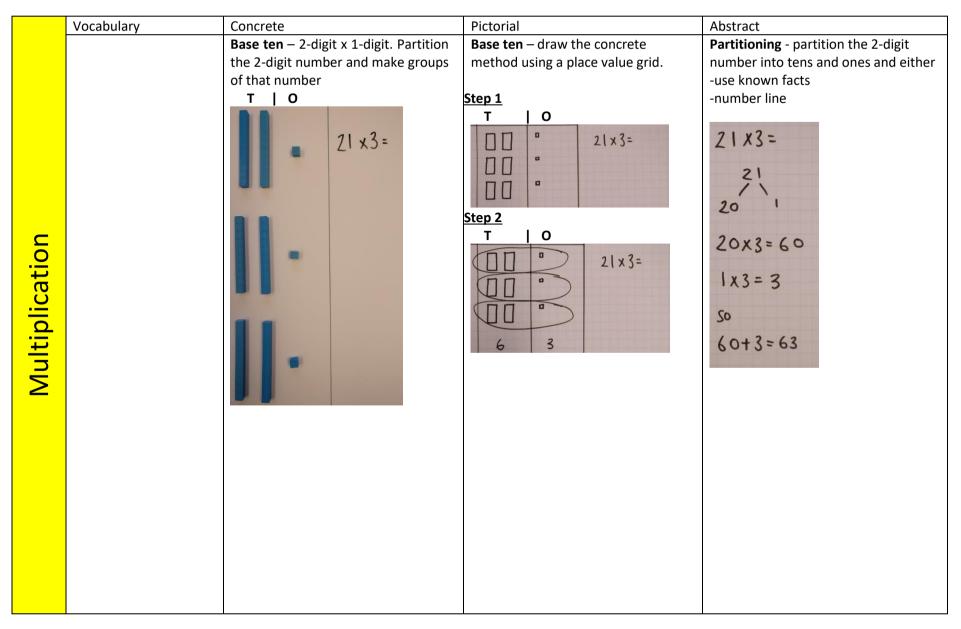
	Vocabulary	Concrete	Pictorial	Abstract
Subtraction			Bar model – parts of a whole - finding the difference between the whole and the part	

Vocabularv Concrete Pictorial Abstract Place value counters – partition and **Place value counters** – draw the Abstract calculation – record the remove the number that is being concrete method. Partition and formal written method eventually subtracted, exchanging when remove the number that is being without the pictorial method necessarv. subtracted, exchanging when alongside. Step 1 necessary. н т 0 Children **must** understand what has Step 1 1 happened when they cross out digits ТІО н 329 329 0000 000 00 0 8 172-Step 1 õ Step 2 ТΙ 0 н 3 2 000 5 00 000 7 Step 2 н ТΙ 0 Step 3 Н | T | O Step 2 28 2 9 329 0000 000 00 8 172 700 000000 7 8 Step 4 н ΤI 0 Step 3 28 2 5 000 000 00 0 н Т 7 2 7000000 57 7 32 8 7

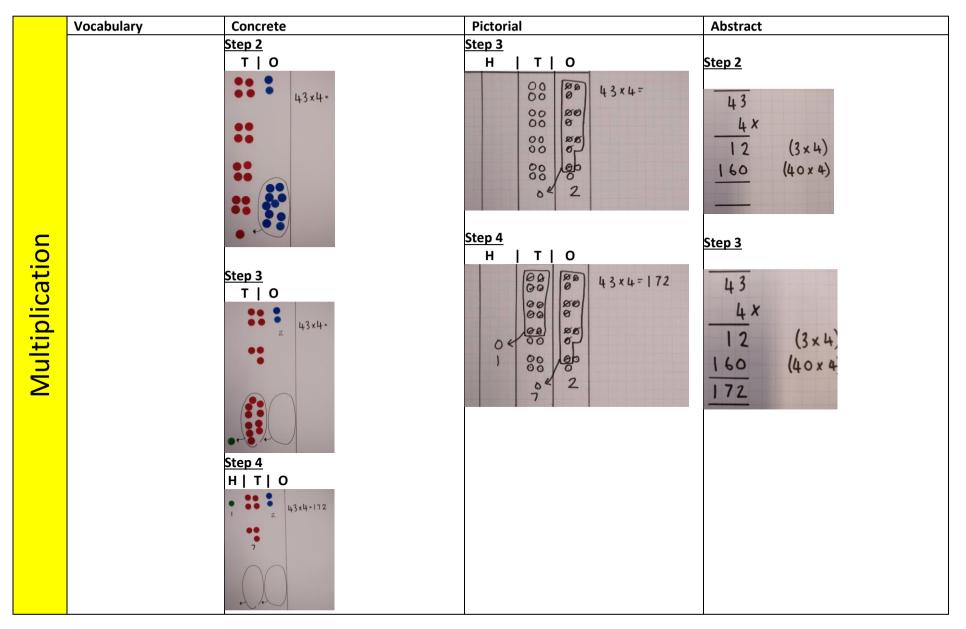


				-
	Vocabulary	Concrete	Pictorial	Abstract
	abstract	Repeated addition – use Numicon	Repeated addition – draw the	Record multiplication alongside
	arrange	and counters.	concrete method.	repeated addition
	array	2 / 1 =	$2 \times 11 -$	
	base ten	774	774-	
	bead strings	00 00 00		
	calculation		(00) (00) (00)	
	combine			
	concrete	4 + 4 + 4		
	counters		11 + 1+ + 4	
C	double	3×4=	4 + + + +	
<u>.0</u>	equal	7.4-		
at	equal groups			
Ŭ	exchange groups of			
ilo	hundreds			
Multiplication	lots of	Repeated addition – use multilink	Repeated addition – make equal	Repeated addition – make equal
n	multilink		jumps along the number line	jumps along an empty number line
5	multiply	$3 \times 4 =$		(ENL)
	numicon	J	3×4=	2 ~ 1.
	number facts			JX4=
	number line number track	$1_{1} + 1_{2} + 1_{4}$	+4 +4 +4	
	ones / units*	4 4 4	0123456789101121314151617181920	+4 +4 +4
	partition	Bead string – group the beads into 4's		
	pictorial	to calculate		0 4 8 12
	place value			
	PPW – part-part-whole	3×4 =		
	product	3.4		
	record			
	repeated addition	4 + 4 + 4		
	tens			

Concrete Pictorial Vocabularv Abstract Arrays – use counters. Arrange the Arrays – draw the concrete method Arrays – use arrays to find multiple groups in rows and columns highlighting commutativity information $3 \times 4 = 4 \times 3$ (000)4×3=12 00) 3+3+3+3=12 000 000 (0000) 3x4=12 0000 4+4+4=12 0000 0000 0000 **Multiplication Peg Board** – use pegs to make the Bar models – make sure all sections array are proportional (also can be used for division)



Vocabularv Concrete Pictorial Abstract Place value counters – 2-digit x 1-Place value counters – draw the Begin formal written method – digit (no exchange). Partition the 2partition the 2-digit number, multiply concrete method using a place value digit number and combine the grid. Partition the 2-digit number and the tens and ones separately and then correct number of equal groups. combine the correct number of equal combine. (Specific coloured counters 2123= groups. represent a place value) Step 1 0 т 21 x3= 71×3= 00 0 00 0 21 0 00 3 * Step 2 т 0 **Multiplication** 1 X 3 $21 \times 3 = 63$ 00 0 20×3=60 0 00 0 00 63 6 2 Place value counters - 2-digit x 1-Place value counters – 2-digit by 1-Formal written method – short digit (with exchange). Partition the 2digit (with exchange) – draw the multiplication (expanded) digit number and combine the concrete method using a place value correct number of equal groups and Step 1 grid. exchange where necessary. Step 1 Step 1 43 TIO Т | О Н 4x 000 00 43×4= 43×4= ... 12 (3×4) 00 00 00 0 00 00 0 000 00



	Vocabulary	Concrete	Pictorial	Abstract
				Short multiplication –
tion				Short multiplication = $\frac{342}{\times 7}$ $\frac{4}{4}$ Step 2
Multiplication				342 × 7 94 21 215/02/04
				Step 3
				342 × 7 2394 21 2018/12/14

	Vocabulary	Concrete	Pictorial	Abstract
				Long multiplication – expanded
				Step 1
				342
				<u>×7</u> 14 2×7
				1
				<u>Step 2</u>
				342
				×Т
				280 40×7
ti				2 30
C				
lic				Step 3
Multiplication				342
- Tr				$-\frac{\times 7}{142\times7}$
Ś				280 40×7
				2100 300 × 7
				Step 4
				342 ×7
				14 2×7
				2100 300×7
				20(1)((3))

	Vocabulary	Concrete	Pictorial	Abstract
				$\frac{5 \text{tep 5}}{342}$ $\frac{342}{28040 \times 7}$ $\frac{2100300 \times 7}{2394}$
				Long multiplication – Compact - 2-digit x 2-digit
Multiplication				Step 1 $\begin{array}{c} \times 1 & 6 \\ 2 & 4 \\ \hline \end{array}$ Step 2 $\begin{array}{c} \times 1 & 6 \\ \hline 1 & 4 \\ \hline \end{array}$ Step 3 $\begin{array}{c} \times 1 & 6 \\ \hline 1 & 4 \\ \hline \end{array}$ $\begin{array}{c} \times 1 & 6 \\ \hline 1 & 4 \\ \hline \end{array}$

	Vocabulary	Concrete	Pictorial	Abstract
Multiplication	Vocabulary	Concrete	Pictorial	Abstract Step 4 24 24 24 24 24 24 24 24 24 24 24 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 384 240 250

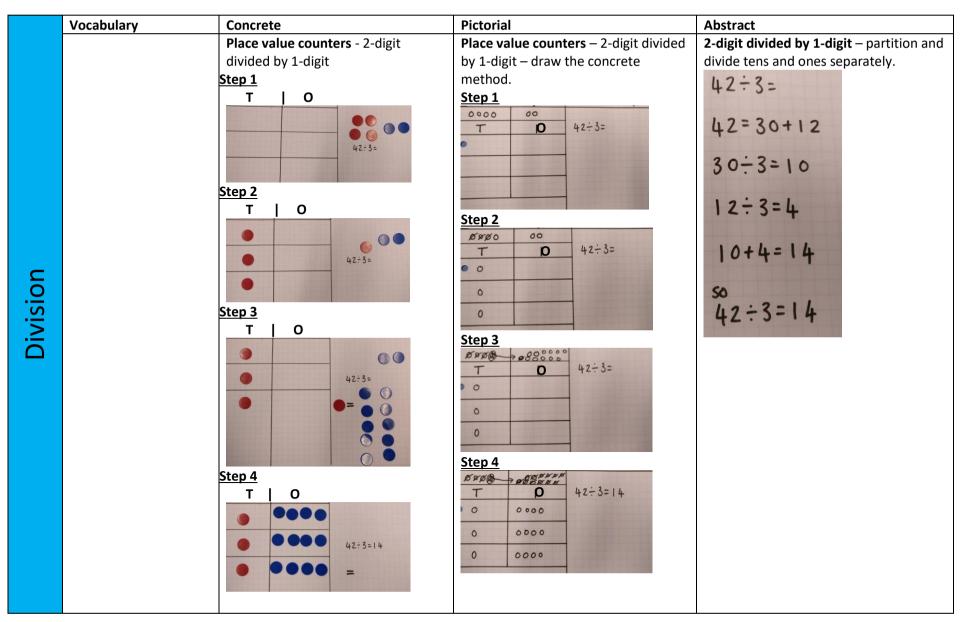
	Vocabulary	Concrete	Pictorial	Abstract
				Step 3
Multiplication				$\frac{124}{26}$ $\frac{724}{724}$ $\frac{124}{726}$ $\frac{124}{726}$ $\frac{724}{724}$ 80
JL				2019, Step 5
2				124 <u>+26</u> <u>744</u> <u>480</u> 2018

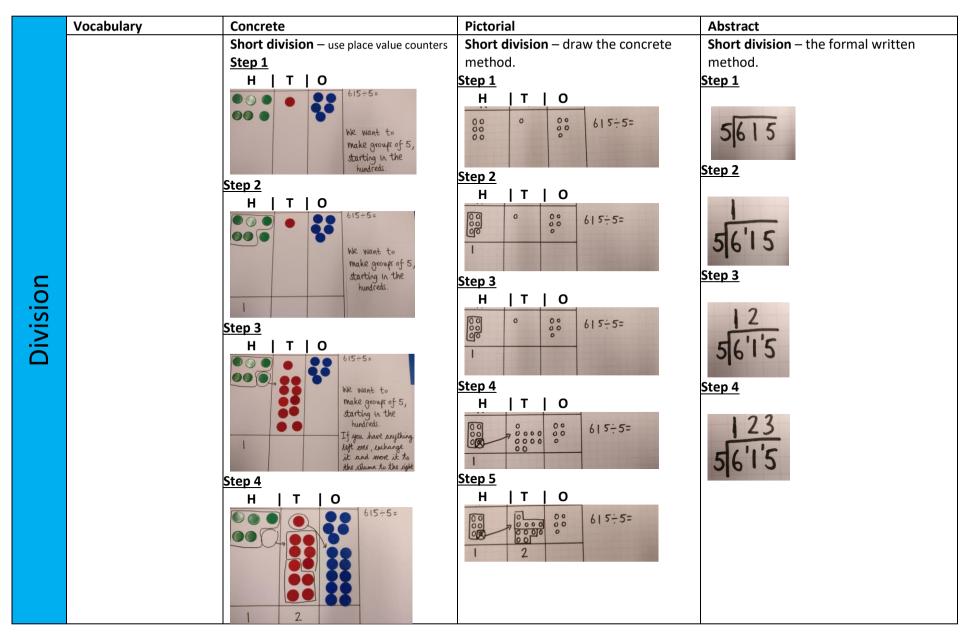
	Vocabulary	Concrete	Pictorial	Abstract
				Step 6
				× 26 744
				2480
				2019/02
Multiplication				Step 7 124 726 724 2480 3224 2019/02/05

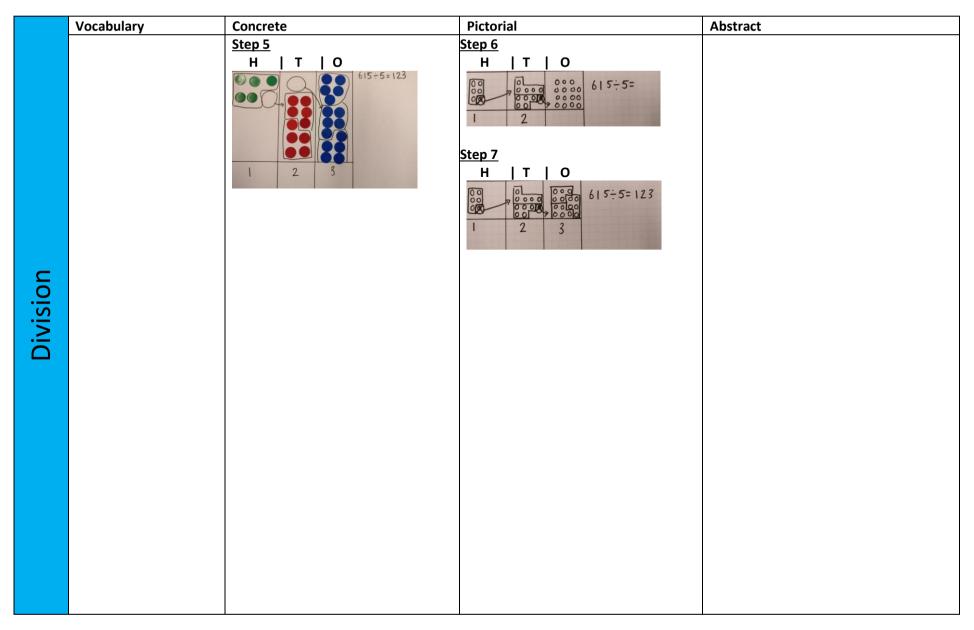
	Vocabulary	Concrete	Pictorial	Abstract		
	abstract	Sharing – share counters into equal	Sharing – draw the concrete method.	Sharing - record division using correct		
	arrange	groups.	Share number into equal groups	symbol.		
	array	Step 1	20 counters shared into 4 equal groups			
	base ten	20 counters shared into 4 equal groups	and the second			
	calculation		the second s	5 00 11 - 5		
	concrete		the second s	³ 20-4 = J		
	counters			(\cdot, \cdot)		
	divide		$ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$			
	division	() () () ()	5 50 50 00	5 5		
	equal					
	equal groups	Chan 2				
_	exchange	Step 2				
ð	grouping					
S.	half					
:5	halve					
Division	hundreds	Sharing - share multilink into equal				
	long division multilink	groups and arrange them in rows				
	numicon	(beginnings of arrays)				
	numicon number facts	Step 1				
	number line	Break the multilish into 4 equal pieces.				
	number track					
	ones/units*					
	partition	Step 2				
	pictorial	Break the multilisk into 4 equal pieces.				
	place value					
	PPW – part-part-whole					
	product					
	record					
	remainder					
	repeated subtraction					

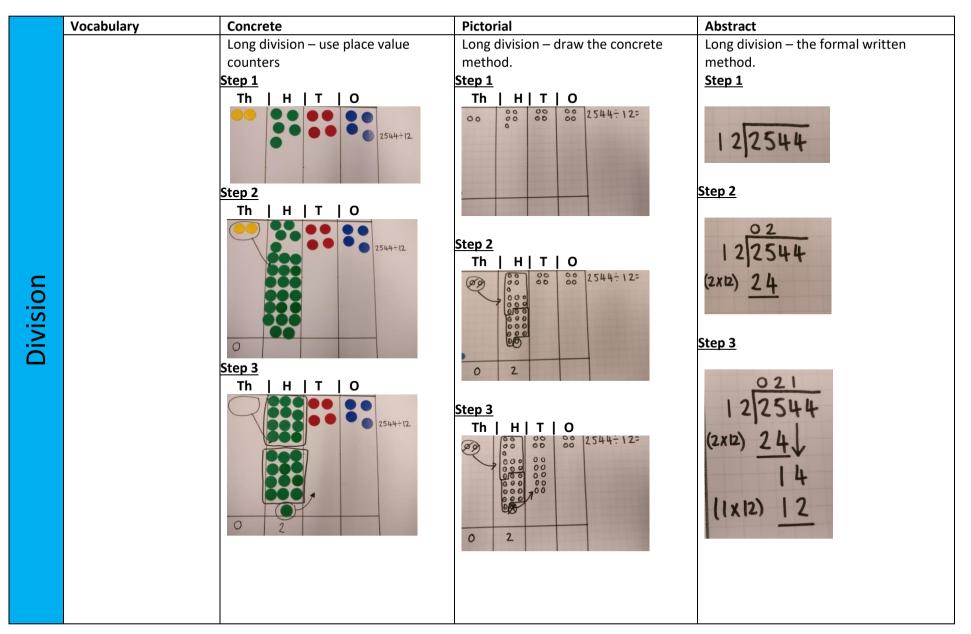
	Vocabulary	Concrete	Pictorial	Abstract
		Grouping – put counters into equal	Grouping – draw the concrete	Abstract calculation – record division
		groups	method.	using correct symbol.
		<u>Step 1</u>	<u>Step 1</u>	<u>Step 1</u>
		Put 20 counter into groups of 4.	Put 20 counters into groups of 4.	20÷ =4
		Step 2	<u>Step 2</u>	<u>Step 2</u>
			Put 20 counters into groups of 4.	
c		Put 20 counter into groups of 4.		
0	Grouping – group counters along a number line or number track.		Grouping – repeated subtraction	Grouping – repeated subtraction along
Division			using counters/multilink along a number line	a number line (marked or empty).
				-4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -4 -2 16 -2
			Bar model – number of equal parts	
			equaling a whole	
			? 4 4 4 4	

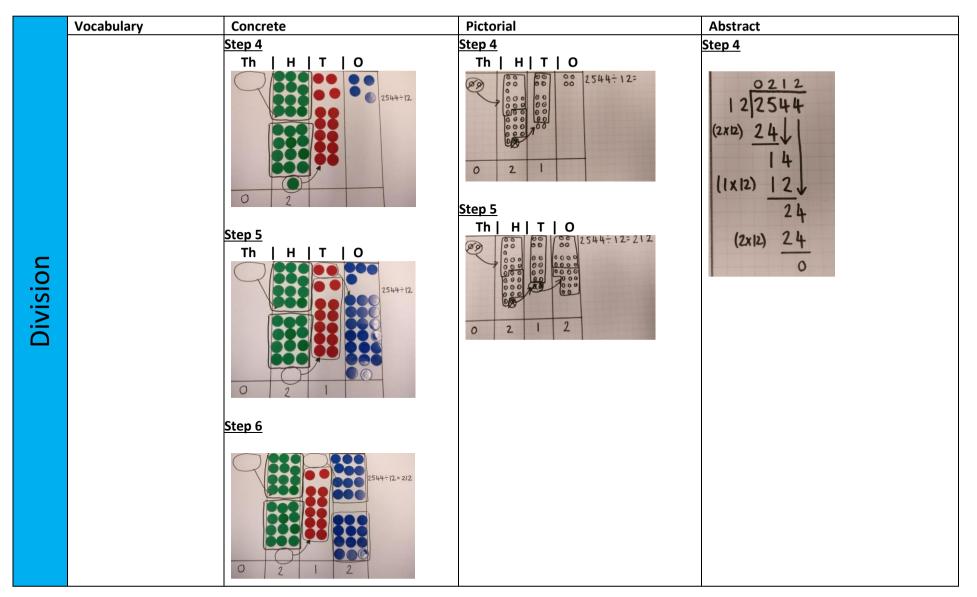
	Vocabulary	Concrete	Pictorial	Abstract
	Vocabulary	Concrete Grouping/sharing with remainder – group or share counters as the question requires. Step 1 Share 7 counters into 3 groups	Grouping/sharing with remainder – draw concrete method. Share 7 counters into 3 groups	Abstract Abstract calculation – record operation in numbers alongside pictorial representation (if necessary). Share 7 counters into 3 groups 0 0 0 0 0 $7 \div 3 = 2$ remainderl $7 \div 3 = 2$ remainderl
Division		Share 7 counters into 3 groups		Abstract calculation – use number facts to recognise when there will be a remainder – 13 ÷ 4, 13 isn't in the 4 times table so there will be a remainder. 12 x 4 = 3, so 13 ÷ 4 = 3r1











	Phase 1		Phase 2		Phase 3	
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Mental +/-	•add and subtract one-digit and two-digit numbers to 20, including zero	 add and subtract numbers using concrete objects, pictorial representations, and mentally, including: TO+O, TO+T, TO+TO and O+O+O show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot 	•add and subtract numbers mentally, including: HTO+O, HTO+T and HTO+H		•add and subtract numbers mentally with increasingly large numbers	•perform mental calculations, including with mixed operations and large numbers
Written +/-			 add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction 	 add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate 	 add and subtract whole numbers with more than 4 digits, including using formal written methods 	
Mental (x/÷)		 calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot 	•write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one- digit numbers, using mental methods	 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers recognise and use factor pairs and commutativity in mental calculations 	 multiply and divide numbers mentally drawing upon known facts multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 	•perform mental calculations, including with mixed operations and large numbers
Written (x/÷)			•Progress to formal written methods calculations as above	•multiply two-digit and three- digit numbers by a one-digit number using formal written layout	 multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context 	 multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context